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Strong Signed Degrees in Single Valued Neutrosophic Signed Graphs

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Abstract—*The concept of SVNS-Graph is a generalization of signed fuzzy and signed intuitionistic fuzzy graph. SVNS-Graph deals with the real world problems when situation of indeterminacy occurs. Three types of degrees are associated in these graphs to deal with real applications. On the basis of these degrees we have certain types of SVNS-Graphs.*
The purpose of this paper is to define different types of signed strong degrees, strong size and strong order of SVNS- Graph. Strong constant SVNS-Graph and totally strong constant SVNS-Graph are also discussed in this paper.

Keywords*: Single valued neutrosophic graph(SVNG), Single valued neutrosophic signed graph(SVNSG), Strong degrees, Signed strong degrees.*

1. INTRODUCTION

The fuzzy set theory was developed by Zadeh[1]. Nagoorgani and Radha [7] defined degree of a vertex, regular and totally regular fuzzy graphs. Smarandache gives the notion of single valued neutrosophic graphs. Karunambigai and Buvaneswari [12] introduced the strong and super strong vertices in intuitionistic fuzzy graphs. And some others discussed in [15] strong degrees in single valued neutrosophic graphs.

Also In 2017, Mehra and Singh [15] introduced the concept of single valued neutrosophic signed graphs, motivated by the notion of single valued neutrosophic graphs we apply the concept of strong degrees on single valued neutrosophic signed graphs.

2. DEFINITIONS

Definition (2.1) SVN Set:

Let X be a space of objects with generic elements denoted by x. A single valued neutrosophic set A (SVNS) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x in X, $T_A(x),I_A(x),F_A(x)\in[0,1]$.

A SVNS A can be written as,

$A=\{(x,T_A(x),I_A(x),F_A(x)):x\in X\}$

Definition (2.2) SVN-Graph:

A single valued neutrosophic graph is a pair G=(A,B), where $A:X \rightarrow[0,1]$ is single valued neutrosophic set in X and $B:X\times X\rightarrow[0,1]$ is single valued neutrosophic relation on X such that

$T_B(x,y) \leq \min[T_A(x),T_A(y)]$

$I_B(x,y) \geq \max[I_A(x),I_A(y)]$

$F_B(x,y) \geq \max[F_A(x),F_A(y)]$ for all $x,y\in X$.

A is called single valued neutrosophic vertex set of G and B is called single valued neutrosophic edge set of G, respectively. Also, B is symmetric single valued relation on A. If B is not symmetric then G=(A,B) is called a single valued neutrosophic directed graph.

2.2 Example

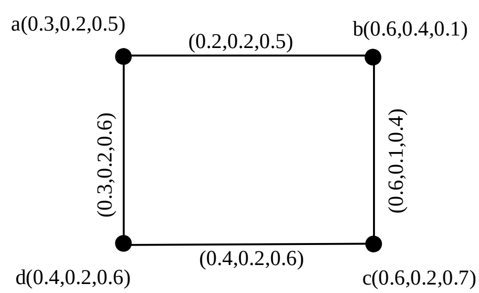


Fig. 1: SVN-Graph

Definition (2.3) SVNS-Graph:

A Single valued neutrosophic graph $S_G=(X,Y)$ is said to be Single valued neutrosophic signed graph(SVNSG) if there is a mapping $\sigma:Y\rightarrow\{+,-\}$ such that each edge assign to {+,-} or all nodes or edges assigned to {+,-}.

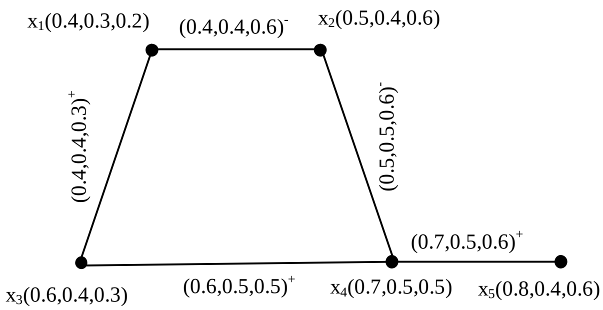


Fig. 2: SVNS-Graph

Definition (2.4) T-strength:-

The T-strength of a path P = x_1x_2…………x_n is defined as min.$\{T_{ij}\}$, for all i,j = 1,2, ………., n and is denoted by s_T.

Definition (2.5) I- strength:-

The I- strength of a path P = x_1x_2…………x_n defined as max.$\{I_{ij}\}$, for all i.j = 1,2,…………, n and is denoted by s_I.

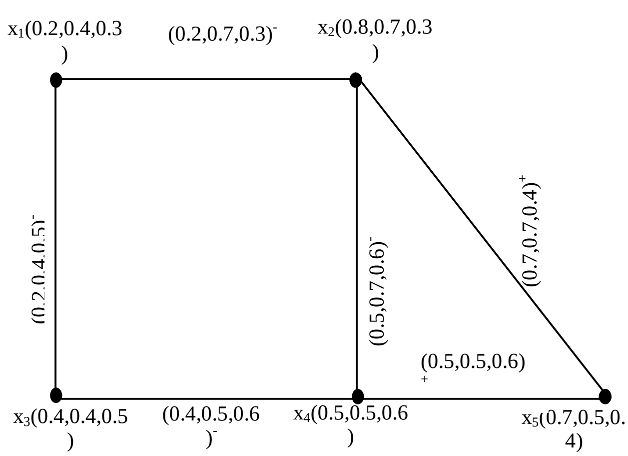
Definition (2.6) F- strength:-

The F- strength of a path P = x_1x_2…………x_n defined as max.$\{F_{ij}\}$, for all i.j = 1,2,…………, n and is denoted by s_F.

Example:-

Consider a balanced single valued Neutrosophic graph

Here,



s_T = T-strength of Path P= $x_1x_2x_4x_5$ is 0.2.

s_I = I- strength of Path P= $x_1x_2x_4x_5$ is 0.7.

s_F = F- strength of Path P= $x_1x_2x_4x_5$ is 0.6.

Note1:- If same edge possesses the values s_T, s_I, s_F, then the value is the strength of the P and is deneted by s_p.

Definition (2.7) T-strength of connectedness:

Let S_G be SVNS-Graph. The T-strength of connectedness between two vertices x_i and x_j is defined as $CONN_{T(S_G)}(x_i, x_j) = max\{s_T\}$.

Definition (2.8) I-strength of connectedness:

Let S_G be SVNS-Graph. The I-strength of connectedness between two vertices x_i and x_j is defined as $CONN_{I(S_G)}(x_i, x_j) = max\{s_I\}$

Definition (2.9) F-strength of connectedness:

Let S_G be SVNS-Graph. The F-strength of connectedness between two vertices x_i and x_j is defined as $CONN_{F(S_G)}(x_i, x_j) = max\{s_F\}$

Note 2:- The notation

$< CONN_{T(S_G)}(x_i, x_j) = max\{s_T\} >$ is used to denote the T-strength of connectedness between x_i and x_j in the SVNS-Graph obtained from S_G by deleting the edge y_{ij}.

Definition(2.10): Let S_G be SVNS-Graph

(i) The signed T-degree of a vertex x_i is

$$Sd_T = \left| \sum_{y_{ij} \in Y} T_{ij}^{+} - \sum_{y_{ij} \in Y} T_{ij}^{-} \right|$$

(ii) The signed I-degree of a vertex x_i is

$$Sd_I = \left| \sum_{y_{ij} \in Y} I_{ij}^{+} - \sum_{y_{ij} \in Y} I_{ij}^{-} \right|$$

(iii) The signed F-degree of a vertex x_i is

$$Sd_F = \left| \sum_{y_{ij} \in Y} F_{ij}^{+} - \sum_{y_{ij} \in Y} F_{ij}^{-} \right|$$

(iv) The signed degree of a vertex x_i is

$Sd(x_i) = <Sd_T, Sd_I. Sd_F>$

$Sd_T(x_1) = |0.3-0.1| = 0.2$

$Sd_I(x_1) = |0.8-0.5| = 0.3$

$Sd_F(x_1) = |0.9-0.4| = 0.5$

Thus, $Sd(x_1) = <0.2, 0.3, 0.5>$

Definition (2.11) Strong edge in SVNS-Graph:
An edge y_{ij} is said to be strong edge in SVNS-Graph S_G if
$T_{ij} \geq CONN_{T(S_G)-y_{ij}}(x_i, x_j)$
$I_{ij} \leq CONN_{I(S_G)-y_{ij}}(x_i, x_j)$
And $F_{ij} \leq CONN_{F(S_G)-y_{ij}}(x_i, x_j)$, For every $(x_i, x_j) \in X$.

Definition (2.12) Weak edge in SVNS-Graph:
An edge y_{ij} is said to be weak edge in SVNS-Graph S_G = (X,Y) if
$T_{ij} < CONN_{T(S_G)-y_{ij}}(x_i, x_j)$
$I_{ij} > CONN_{I(S_G)-y_{ij}}(x_i, x_j)$

And $F_{ij} > CONN_{F(S_G)-y_{ij}}(x_i, x_j)$, For every $(x_i, x_j) \in X$.

Definition (2.13) Strong Vertex:
Let S_G be a SVNS-Graph. A Vertex $x_i \in X$ is said to be strong. If y_{ij} is a strong edge, for all x_j incident with x_i.

Definition (2.14) Signed T-strong degree:
Let S_G = (X,Y) be single valued neutrosophic signed graph. The signed T-strong degree of a vertex $x_i \in X$ is defined as

$$Sd_{s(T)(x_i)} = \left| \sum_{y_{ij}\in Y} T_{ij}^{+} - \sum_{y_{ij}\in Y} T_{ij}^{-} \right|$$

Where y_{ij} are strong edges incident at x_i.

Definition (2.15) Signed I-strong degree:
Let S_G = (X,Y) be single valued neutrosophic signed graph. The signed I-strong degree of a vertex $x_i \in X$ is defined as

$$Sd_{s(I)(x_i)} = \left| \sum_{y_{ij}\in Y} I_{ij}^{+} - \sum_{y_{ij}\in Y} I_{ij}^{-} \right|$$

Where y_{ij} are strong edges incident at x_i.

Definition (2.16) Signed F-strong degree:
Let S_G = (X,Y) be single valued neutrosophic signed graph. The signed F-strong degree of a vertex $x_i \in X$ is defined as

$$Sd_{s(F)(x_i)} = \left| \sum_{y_{ij}\in Y} F_{ij}^{+} - \sum_{y_{ij}\in Y} F_{ij}^{-} \right|$$

Where y_{ij} are strong edges incident at x_i.

Definition (2.17) Signed strong degree of a vertex:
Let S_G = (X,Y) be single valued neutrosophic signed graph. The signed strong degree of a vertex $x_i \in X$ is given by
$Sd_s(x_i) = \langle Sd_{s(T)}(x_i), Sds_{(I)}(x_i), Sds_{(F)}(x_i) \rangle$.

Definition (2.18) Minimum signed strong degree:
Let S_G = (X,Y) be single valued neutrosophic signed graph. The minimum signed strong degree of S_G is defined as
$S\delta_s(S_G) = (S\delta_{S(T)}(S_G), S\delta_{s(I)}(S_G), S\delta_{s(F)}(S_G))$.
Where

$S\delta_{S(T)}(S_G) = \wedge\{Sd_{S(T)}(x_i) \mid x_i \in X\}$ is the minimum signed T-strong degree of S_G.

$S\delta_{S(I)}(S_G) = \wedge\{Sd_{S(I)}(x_i) \mid x_i \in X\}$ is the minimum signed I-strong degree of S_G.

And $S\delta_{S(F)}(S_G) = \wedge\{Sd_{S(F)}(x_i) \mid x_i \in X\}$ is the minimum signed F-strong degree of S_G.

Definition (2.19) Maximum signed strong degree:
Let S_G = (X,Y) be single valued neutosophic signed graph. The maximum signed strong degree of S_G is defined as
$S\Delta_S(S_G) = (S\Delta_{S(T)}(S_G), S\Delta_{S(I)}(S_G), S\Delta_{S(F)}(S_G))$
Where

$S\Delta_{S(T)}(S_G) = \vee\{Sd_{S(T)}(x_i) \mid x_i \in X\}$ is the maximum signed T-strong degree of S_G.

$S\Delta_{S(I)}(S_G) = \vee\{Sd_{S(I)}(x_i) \mid x_i \in X\}$ is the maximum signed I-strong degree of S_G.

$S\Delta_{S(F)}(S_G) = \vee\{Sd_{S(F)}(x_i) \mid x_i \in X\}$ is the maximum signed F-strong degree of S_G.

Definition (2.20) Signed T-total strong degree:
Let S_G = (X,Y) be a SVNS-Graph. The signed T-total strong degree of a vertex $x_i \in X$ in S_G is defined as $Sigtd_{S(T)}(x_i) = Sd_{S(T)}(x_i) + T_i$.

Definition (2.21) Signed F-total strong degree:
Let S_G = (X,Y) be a SVNS-Graph. The signed F-total strong degree of a vertex $x_i \in X$ in S_G is defined as $Sigtd_{S(F)}(x_i) = Sd_{S(F)}(x_i) + F_i$.

Definition (2.22) Signed T-strong size of SVNS – Graph:
Let S_G be a SVNS- Graph. The T-Strong size of S_G is defined as

$$Z_{s(T)(S_G)} = \left| \sum_{x_i\neq x_j} T_{ij}^{+} - \sum_{x_i\neq x_j} T_{ij}^{-} \right|$$

where T_{ij}^{+} and T_{ij}^{-} is the membership of strong edge $y_{ij} \in Y$.

Definition (2.23) Signed I-strong size of SVNS – Graph:
Let S_G be a SVNS- Graph. The I-Strong size of S_G is defined as

$$Z_{s(I)(S_G)} = \left| \sum_{x_i\neq x_j} I_{ij}^{+} - \sum_{x_i\neq x_j} I_{ij}^{-} \right|$$

where I_{ij}^{+} and I_{ij}^{-} is the membership of strong edge $y_{ij} \in Y$.

Definition (2.24) Signed F-strong size of SVNS – Graph:
Let S_G be a SVNS- Graph. The F-Strong size of S_G is defined as

$$Z_{s(F)(S_G)} = \left| \sum_{x_i\neq x_j} F_{ij}^{+} - \sum_{x_i\neq x_j} F_{ij}^{-} \right|$$

Where F_{ij}^{+} and F_{ij}^{-} is the membership of strong edge $y_{ij} \in Y$.

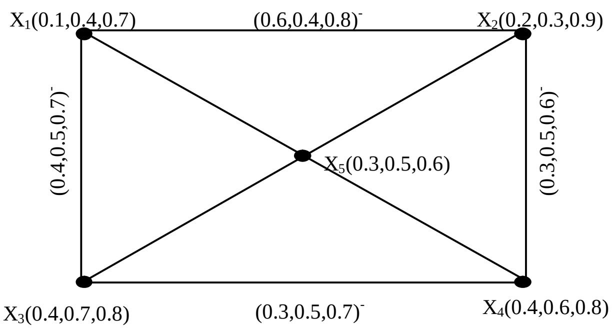
Definition (2.25) Strong size of SVNS- Graph:
Let S_G be a SVNS- Graph. The Strong size of S_G is defined as
$Z_S(S_G) = [Z_{S(T)}(S_G), Z_{S(I)}(S_G), Z_{S(F)}(S_G)]$

Definition (2.26) T – Strong order of SVNS-Graph:
Given the SVNS-Graph S_G=(X,Y). The T-strong order of a SVNS-Graph is defined $O_{S(T)}(S_G) = \left|\sum_{x_i\in X} T_i^{+} - \sum_{x_i\in X} T_i^{-}\right|$ where x_i is the strong vertex in S_G.

Definition (2.27) F – Strong order of SVNS-Graph:
Given the SVNS-Graph S_G = (X,Y). The F-strong order of a SVNS-Graph is defined as

$O_{S(T)}(S_G) = \left|\sum_{x_i \in X} F_i^+ - \sum_{x_i \in X} F_i^-\right|$ where x_i is the strong vertex in S_G.

Definition (2.28) Strong order of SVNS - Graph :
Given the SVNS – Graph $S_G = (X,Y)$. The strong order of a SVNS – Graph is defined as
$O_{S(T)}(S_G) = [O_{s(T)}(S_G), O_{s(I)}(S_G), O_{s(F)}(S_G)]$
Example:-



(i) y_{12}
$(s_{T_1}, s_{I_1}, s_{F_1})$ = (0.2, 0.5, 0.7)
$(s_{T_2}, s_{I_2}, s_{F_2})$ = (0.3, 0.5, 0.6)
$CONN_{(S_G)-y_{12}}(x_1, x_2)$ = (0.3, 0.5, 0.6)
y_{12}(0.6, 0.4, 0.8)
Since T_{12} = 0.6>0.3
I_{12} =0.4<0.5
F_{12} =0.8>0.6
Hence, y_{12} is not a strong edge.

(ii) y_{13}
$(s_{T_1}, s_{I_1}, s_{F_1})$ = (0.2, 0.4, 0.8)
$(s_{T_3}, s_{I_3}, s_{F_3})$ = (0.3, 0.5, 0.7)
$CONN_{(S_G)-y_{13}}(x_1, x_3)$ = (0.3, 0.4, 0.7)
y_{13} (0.4, 0.5, 0.7)
Since T_{13} = 0.4>0.3
I_{13} = 0.5>0.4
F_{13} = 0.7=0.7
Hence, y_{13} is not a strong edge.

(iii) y_{15}
$(s_{T_1}, s_{I_1}, s_{F_1})$ = (0.4, 0.5, 0.8)
$(s_{T_5}, s_{I_5}, s_{F_5})$ = (0.7, 0.5, 0.6)
$CONN_{(S_G)-y_{15}}(x_1, x_5)$ =
(0.7, 0.5,0.6)
y_{15} (0.2, 0.3, 0.4)
Since T_{15} =0.2<0.7
Hence, y_{15} is not a strong edge.

(iv) y_{24}
$(s_{T_2}, s_{I_2}, s_{F_2})$ = (0.6, 0.4, 0.8)
$(s_{T_4}, s_{I_4}, s_{F_4})$ = (0.3, 0.5, 0.7)
$CONN_{(S_G)-y_{24}}(x_2, x_4)$ =
(0.6, 0.4, 0.7)
y_{24} (0.3, 0.5, 0.6)
Since T_{24} =0.3<0.6
Hence, y_{24} is not a strong edge.

(v) y_{25}
$(s_{T_2}, s_{I_2}, s_{F_2})$ = (0.3, 0.5, 0.8)
$(s_{T_5}, s_{I_5}, s_{F_5})$ = (0.2, 0.5, 0.6)
$CONN_{(S_G)-y_{25}}(x_2, x_5)$ =
(0.3, 0.5, 0.6)
y_{25} (0.7, 0.2, 0.4)
Since T_{25} =0.7>0.3
I_{25} =0.2<0.5
F_{25} =0.4<0.6
Hence, y_{25} is a strong edge.

(vi) y_{35}
$(s_{T_3}, s_{I_3}, s_{F_3})$ = (0.3, 0.5, 0.7)
$(s_{T_5}, s_{I_5}, s_{F_5})$ = (0.2, 0.3, 0.4)
$CONN_{(S_G)-y_{35}}(x_3, x_5)$ =
(0.3, 0.3, 0.4)
y_{35} (0.8, 0.5, 0.6)
Since T_{35} = 0.8>0.3
F_{35} =0.5>0.3
Hence, y_{35} is not a strong edge.

(vii) y_{34}
$(s_{T_3}, s_{I_3}, s_{F_3})$ = (0.4, 0.5, 0.7)
$(s_{T_4}, s_{I_4}, s_{F_4})$ = (0.3, 0.5, 0.6)
$CONN_{(S_G)-y_{34}}(x_3, x_4)$ = (0.4, 0.5, 0.6)
y_{34} (0.3, 0.5, 0.7)
Since T_{34} =0.3<0.4
Hence, y_{34} is not a strong edge.

(viii) y_{45}
$(s_{T_4}, s_{I_4}, s_{F_4})$ = (0.3, 0.5, 0.7)
$(s_{T_5}, s_{I_5}, s_{F_5})$ = (0.3, 0.5, 0.7)
CONN $(S_G) - y_{45}(x_4,x_5)$ =
(0.3,0.5,0.6)
y_{45} (0.7,0.3,0.2)
Since T_{45} =0.7>0.3
I_{45} =0.3<0.5
F_{45} =0.2<0.6
Hence,y_{45} is a strong edge.

Therefore, edges y_{25} and y_{45} are strong. Now,
Sdeg (x_1) = (0,0,0)
Sdeg (x_2) = (0.7,0.2,0.4)
Sdeg (x_3) = (0,0,0)
Sdeg (x_4) = (0.7,0.3,0.2)
Sdeg (x_5) = [(0.7,0.2,0.4) + (0.7,0.3,0.2)] = (1.4,0.5,0.6)
Vertices x_1 and x_3 have signed degree (0,0,0)
Because they not have strong edge incident on it.

Example :-

Consider the SVNS – Graph given in above example, we have that y_{25} and y_{45} are strong edges in this graph.
If $Z_s(S_G)$ denotes strong size of SVNS – Graph then
$Z_{s(T)}(S_G)$ = 0.7 + 0.7 = 1.4
$Z_{s(I)}(S_G)$ = 0.2 + 0.3 = 0.5
$Z_{s(F)}(S_G)$ = 0.4 + 0.2 = 0.6

Therefore, $Z_s(S_G)$ = (1.4,0.5,0.6).

Example :-

In the Graph S_G given in above example, the strong order is zero.

O_s (S_G) = (0,0,0)

Because, in this Graph there is no strong vertex.

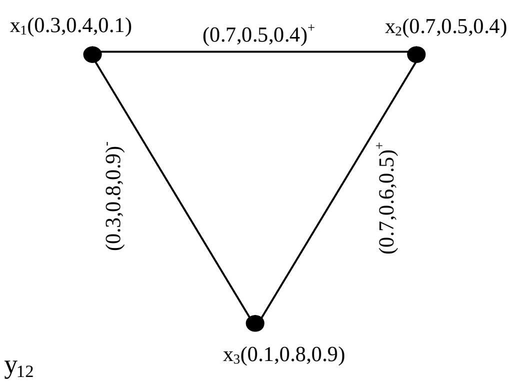
Definition (2.29)Strong constant SVNS – Graphs:

Let S_G be a SVNS – Graphs. If $Sd_{s(T)}$ (x_i) = k_1, $sd_{s(I)}$ (x_i) = k_2, $Sd_{s(F)}$ (x_i) = k_3 for all $x_i \in X$, then the SVNS – Graphs is called as (k_1,k_2,k_3) – strong constant SVNS -Graphs, Strong constant SVNS – Graphs of degree (k_1,k_2,k_3).

Definition (2.30)Totally Strong constant SVNS – Graphs:

Let S_G be a SVNS – Graphs. If $sigtd_{s(T)}$ (x_i) = r_1, $Sigt_{s(I)}$ (x_i) = r_2, $Sigt_{s(F)}$ (x_i) = r_3 for all $x_i \in X$,them the SVNS –Graph is called as (r_1, r_2 ,r_3) – totally strong constant SVNS – Graph of degree (r_1,r_2,r_3).

Example:-



(i) y_{12}

$(s_{T_1}, s_{I_1}, s_{F_1})$ = (0.3, 0.8, 0.9)

$(s_{T_2}, s_{I_2}, s_{F_2})$ = (0.7, 0.6, 0.5)

$CONN_{(S_G)-y_{12}}$ (x_1, x_2) = (0.7, 0.6, 0.5)

Also, y_{12} (0.7, 0.5, 0.4)

Since T_{12} =0.7=0.7

I_{12} =0.5<0.6

F_{12} =0.4<0.5

This implies y_{12} is a strong edge.

(ii) y_{13}

$(s_{T_1}, s_{I_1}, s_{F_1})$ = (0.7, 0.5, 0.4)

$(s_{T_3}, s_{I_3}, s_{F_3})$ = (0.7, 0.6, 0.5)

$CONN_{(S_G)-y_{13}}$ (x_1, x_3) = (0.7, 0.5,0.4)

Also, y_{13} (0.3, 0.8, 0.9)

Since T_{13} = 0.3<0.7

This implies y_{13} is not a strong edge.

(iii) y_{23}

$(s_{T_2}, s_{I_2}, s_{F_2})$ = (0.7, 0.5, 0.4)

$(s_{T_3}, s_{I_3}, s_{F_3})$ = (0.3, 0.8, 0.9)

$CONN_{(S_G)-y_{23}}$ (x_2, x_3) = (0.7, 0.5, 0.4)

y_{23} (0.7, 0.6, 0.5)

Since T_{23} =0.7=0.7

I_{23} =0.6>0.5

This implies y_{23} is not a strong edge.

Now,

$Sdeg_s(x_1)$ = (0.7, 0.5, 0.4)

$Sdeg_s(x_2)$ = (0.7, 0.5, 0.4)

$Sdeg_s(x_3)$ = (0, 0, 0)

Signed strong degree of vertex x_3 is (0,0,0) because no strong edge incident on it.

3. Preliminaries

Proposition (3.1):

In a connected SVNS- Graph

$$2^{Z_{s(T)}(S_G)} = \sum_{i=1}^{n} Sd_{s(T)}(x_i)$$

$$2^{Z_{s(I)}(S_G)} = \sum_{i=1}^{n} Sd_{s(I)}(x_i)$$

$$2^{Z_{s(F)}(S_G)} = \sum_{i=1}^{n} Sd_{s(F)}(x_i)$$

Proposition (3.2) :

In a connected SVNS – Graphs ,

(1) $Sd_{s(T)}$ (x_i) ≤ Sd_{Ti} , $Sd_{s(I)}$ (x_i) ≤ Sd_{Ii} and $Sd_{s(F)}$ (x_i) ≤ Sd_{Fi}.

(2) $Sigtd_{s(T)}$ (x_i) ≤ $Sigtd_{Ti}$, $Sigtd_{s(I)}$ (x_i) ≤ $Sigtd_{Ii}$ and $Sigtd_{s(F)}$ (x_i) ≤ $Sigtd_{Fi}$.

Proposition (3.3)

Let S_G be a SVNS – Graphs where crisp graph S_{G^*} is an odd cycle. Then, S_G is strong constant if f < T_{ij}, I_{ij}, F_{ij} > is constant function for every $y_{ij} \in Y$.

Proposition (3.4)

Let S_G be a SVNS – Graphs where crisp graph S_{G^*} is an even cycle. Then, S_G is strong constant if f < T_{ij}, I_{ij}, F_{ij} > is constant function or alternate edges have same true membership, Indeterminacy membership and false membership for every $y_{ij} \in Y$.

Remark (3.1)

Both of the above proposition 3.3 and 3.4 held for totally strong constant SVNS – Graph, if < T_i, I_i, F_i > is a constant function.

Remark (3.2)
A complete SVNS – Graph need not be a strong constant SVNS – Graph and totally strong constant SVNS – Graph.

Remark (3.3)
A strong SVNS – Graph need not be a strong constant SVNS – Graph and totally strong constant SVNS – Graph.

Remarks (3.4)
For a strong vertex $x_i \in X$,

(i) $Sd_T(x_i) = Sd_{ST}(x_i)$, $Sd_I(x_i) = Sd_{SI}(x_i)$ and $Sd_F(x_i) = Sd_{SF}(x_i)$

(ii) $Sigtd_T(x_i) = Sigtd_{ST}(x_i)$, $Sigtd_I(x_i) = Sigtd_{SI}(x_i)$ and $Sigtd_F(x_i) = Sigtd_{SF}(x_i)$

Theorem (3.1)
Let S_G be a complete SVNS-Graph with $X=\{x_1, x_2, \ldots, x_n\}$ such that $ST_1 \le ST_2 \le ST_3 \le \ldots \le ST_n$, $SI_1 \ge SI_2 \ge SI_3 \ge \ldots \ge SI_n$.
And $SF_1 \ge SF_2 \ge SF_3 \ge \ldots \ge SF_n$. Then,

1. ST_{1j} is minimum signed edge truth membership, SI_{1j} is the maximum signed edge indeterminacy membership and SF_{1j} is the maximum signed edge falsity membership of y_{ij} emits from x_1 for all j = 2,3, …………, n.
2. ST_{in} is maximum signed edge truth membership, SI_{in} is the minimum signed edge indeterminacy membership and SF_{in} is the minimum signed edge falsity membership of among all edges from emits from x_i to x_n for all i= 1, 2, 3, 4, ………., n-1,
3. $Sigtd_t(x_1) = S\Delta td_T(S_G) = n.T_1$,
   $Sigtd_I(x_1) = S\Delta td_I(S_G) = n.I_1$, and
   $Sigtd_F(x_1) = S\Delta td_F(S_G) = n.F_1$
4. $Sigtd_T(x_n)=S\Delta_{tdT}(S_G) = \sum_{i=1}^{n} ST_i$

   $Sigtd_I(x_n) = S\delta_{tdI}(S_G) = \sum_{i=1}^{n} SI_i$ and

   $Sigtd_F(x_n) = S\delta_{tdF}(S_G) = \sum_{i=1}^{n} SF_i$

Proof:- In this proof, throughout suppose that
$ST_1 \le ST_2 \le ST_3 \le \ldots \le ST_n$,
$SI_1 \ge SI_2 \ge SI_3 \ge \ldots \ge SI_n$ and
$SF_1 \ge SF_2 \ge SF_3 \ge \ldots \ge SF_n$.

1. To prove that ST_{1j} is minimum signed edge truth membership, SI_{1j} is the maximum signed edge indeterminacy membership and SF_{ij} is the maximum signed edge falsity membership of y_{ij} emits from $x_1 \forall$ j = 2,3, …..n.
   Assume the contrary i.e, $y_{1\ell}$ is not an edge of minimum signed true membership, maximum signed indeterminate membership and maximum signed false membership emits from x_ℓ.
   Also, let y_{kl}, $2 \le k \le n$, $k \ne 1$ be an edge with minimum signed true membership, maximum signed indeterminate membership and maximum signed false membership emits from y_k.
   Being a complete SVNS-Graph,
   $T_{1\ell} = \min. \{ST_1, ST_\ell\}$, $SI_{1\ell} = \max\{I_1, I_\ell\}$ and
   $SF_{1\ell} = \max\{SF_1, SF_\ell\}$
   Then, $ST_{k\ell} = \min\{ST_k, ST_\ell\}$, $SI_{k\ell} = \max\{SI_k, SI_\ell\}$
   And $SF_{k\ell} = \max\{SF_k, SF_\ell\}$

Since,

$$ST_{k\ell} < ST_{1\ell} \Rightarrow \min\{ST_k, ST_\ell\} < \min\{ST_1, ST_\ell\}$$

Thus either $ST_k < ST_1$ or $ST_\ell < ST_1$.

Also, since $SI_{k\ell} > SI_{1\ell} \Rightarrow \max.\{SI_k, SI_\ell\} > \max\{SI_1, SI_\ell\}$

So either $SI_k > SI_1$, or $SI_\ell > SI_1$.

Since $k \ne 1$, this is contradiction to our vertex assumption that ST_1 is the unique minimum signed vertex true membership, SI_1 is the maximum signed vertex indeterminacy membership and SF_1 is the maximum signed vertex false membership.

Hence, ST_{1j} is minimum signed edge true membership, SI_{ij} is the maximum signed edge indeterminate membership and FI_{1j} is the maximum signed edge false membership of y_{ij} emits from x_1 to x_j for all j = 2, 3, 4,……,n.

2. On the contrary, assume that y_{kn} is not an edge with maximum signed true membership and minimum signed false membership emits from x_k for $1 \le k \le n-1$.

On the other hand, let y_{kr} be an edge with maximum signed true membership, minimum signed indeterminate membership and minimum signed false membership emits from x_r from $1 \le r \le n-1$, $k \ne r$.

Then, $ST_{kr} > ST_{kn} \Rightarrow \min\{ST_k, ST_r\} > \min\{ST_k, ST_n\}$

$= ST_k$, so $ST_r > ST_k$.

$SI_{kr} < SI_{kn} \Rightarrow \max\{SI_k, SI_n\} = SI_k$, so $SI_r < SI_k$

And

Similarly, $SF_{kr} < SF_{kn} \Rightarrow \max\{SF_k, SF_r\} < \max\{SF_k, SF_n\} = SF_k \Rightarrow SF_r < SF_k$.

So $ST_{kr} = ST_k = ST_{kn}$, $SI_{kr} = SI_k = SI_{kn}$ and

$SF_{kr} = SF_k = SF_{kn}$, which is a contradiction.

Hence y_{kn} is an edge with maximum signed true membership, minimum signed indeterminate membership and minimum signed membership among all edges emits from x_k to x_n.

3. Now,

$$sigtd_T(x_1) = Sd_T(x_1) + ST_1$$

$$= \sum_{y_{ij} \in y} ST_{1j} + ST_1 = \sum_{j=2}^{n} ST_{1j} + ST_1$$

$= (n-1)\, ST_1 + ST_1 = nST_1 - ST_1 + ST_1 = nT_1$,

$Sigtd_I(x_1) = Sd_I(x_1) + SI_1$

$$= \sum_{y_{ij} \in y} SI_{1j} + SI_1 = \sum_{j=2}^{n} I_{1j} + SI_1$$

= (n-1) SI_1+SI_1 = nSI_1- SI_1+ SI_1 = nSI_1 and
Similarly,
$Sigt_F(x_1) = Sd_F(x_1) + SF_1$

$$\sum_{y_{ij} \in Y} SF_{1j} + SF_1 = \sum_{j=2}^{n} SF_{1j} + SF_1$$

= (n-1) SF_1 + SF_1 = nSF_1- SF_{1+} SF_1 = nSF_1

Suppose that $Sigtd_T(x_1) \neq S\delta_{tdT}(S_G)$ and let x_k, $k \neq 1$ be a vertex in S_G with minimum signed T-total degree.
Then, Sig $td_T(x_1)$ > $sigtd_T(x_k)$

$$\Rightarrow \sum_{i=2}^{n} ST_{1i} + ST_1 > \sum_{k \neq 1, k \neq j} ST_{kj} + ST_k$$

$$\Rightarrow \sum_{i=2}^{n} ST_1 \wedge ST_i + ST_1 > \sum_{k \neq 1, k \neq j} ST_k \wedge ST_j + ST_k$$

Since $ST_1 \wedge ST_i$ for i=1,2, ……….., n and for all other indices j, $ST_k \wedge ST_j > ST_1$, it follow that

$$(n-1)\, ST_1 + ST_1 > \sum_{k \neq 1, k \neq j} ST_k \wedge ST_j + ST_k > (n-1)\, ST_1 + ST_1$$

Hence, $Sigtd_T(x_1) > \delta igtd_T(x_1)$, which is a contradiction.

Therefore, $Sigtd_T(x_1) = S\delta td_T(S_G)$.

Suppose that

$Sigtd_I(x_1) \neq Sigtd_I(S_G)$ and let x_k, $k\neq1$ be a vertex in S_G with maximum signed I-total degree.

Then, $Sigtd_1(x_1) < Sigtd_I(x_k)$

$$\Rightarrow \sum_{i=2}^{n} SI_{1j} + SI_1 < \sum_{k \neq 1, k \neq j} SI_{kj} + SI_k$$

$$\Rightarrow \sum_{i=2}^{n} SI_1 \vee SI_i + SI_1 < \sum_{k \neq 1, k \neq j} SI_k \vee SI_j + SI_k$$

Since $SI_1 \vee SI_i = SI_1$ for i=1,2,3, ……., n and for all other indices, j, $SI_k \vee SI_j < SI_1$, it follows that

$$(n-1)\, SI_1 + SI_1 < \sum_{k \neq 1, k \neq j} SI_k \vee SI_j + SI_k < (n-1)\, SI_1 + SI_1$$

So that $Sigtd_I(x_1) < Sigtd_I(x_1)$, a contradiction

Therefore, $Sigtd_F(x_1) = Sig\Delta t_{d_F}(S_G)$.
Hence,
$Sigtd(x_1) = S\delta td_T(S_G) = n.ST_1$,
$Sigtd_I(x_1) = S\Delta td_I(S_G) = n.SI_1$ and
$Sigtd_F(x_1) = S\Delta td_F(S_G) = n.SF_1$
Since,
$ST_n > ST_i$, $SI_n < SI_i$ and $SF_n < SF_i$, i=1,2,………….., n-1 and S_G is complete.
$ST_{ni} = ST_n \wedge ST_i = ST_i$,
$SI_{ni} = SI_n \vee SI_i = SI_i$ and
$SF_{ni} = SF_n \vee SF_i = SF_i$
Hence,

$$Sigtd_T(x_n) = \sum_{i=1}^{n-1} ST_i + ST_n$$

$$= \sum_{i=1}^{n-1} (ST_n \wedge ST_i) + ST_n$$

$$= \sum_{i=1}^{n-1} ST_i + ST_n$$

$$= \sum_{i=1}^{n} ST_i$$

$$Sigtd_I(x_n) = \sum_{i=1}^{n-1} SI_{ni} + SI_n$$

$$= \sum_{i=1}^{n-1} (SI_n \vee SI_i) + SI_n$$

$$= \sum_{i=1}^{n-1} SI_i + SI_n$$

$$= \sum_{i=1}^{n} SI_i$$

Suppose that $Sigtd_T(x_n) \neq S\Delta td_T(S_G)$. let x_i, $1 \leq \ell \leq n-1$ be a vertex in G such that

$Sigtd_T(x_\ell) = S\Delta td_T(S_G)$ and

$Sigtd_T(x_n) < Sigtd_T(x_\ell)$. In addition,

$$Sigtd_T(x_\ell) = \left[\sum_{i=1}^{\ell-1} ST_{i\ell} + \sum_{i=\ell+1}^{n-1} ST_{i\ell} + ST_{n\ell}\right] + ST_\ell$$

$$\leq \left[\sum_{i=1}^{\ell-1} ST_i + (n-1)ST_{n\ell} + ST_\ell\right] + ST_\ell$$

$$\leq \sum_{i=1}^{n} ST_i = Sigtd_T(x_n)$$

Thus, $Sigtd_T(x_n) \geq Sigtd_T(x_\ell)$,

This is a contradiction.

So, $Sigtd_T(x_n) = S\Delta td_T(S_G) = \sum_{i=1}^{n} ST_i$.

Suppose that $Sigtd_I(x_n) \neq S\delta td_1(S_G)$.

Let x_i, $1 \leq \ell \leq n-1$ be a vertex in S_G such that

$Sigtd_I(x_\ell) = S\delta td_1(S_G)$ and $Sigtd_I(x_n) > Sigtd_1(x_\ell)$

In addition,

$$Sigtd_I(x_\ell) = \left[\sum_{i=1}^{\ell-1} SI_{i\ell} + \sum_{i=\ell+1}^{n-1} SI_{i\ell} + SI_{n\ell}\right] + SI_\ell$$

$$\geq \left[\sum_{i=1}^{\ell-1} SI_i + (n-1)SI_{n\ell} + SI_\ell\right] + SI_\ell$$

$$\geq \sum_{i=1}^{n} ST_i + SI_\ell$$

$$\leq \sum_{i=1}^{n} SI_i = Sigtd_\ell(x_n)$$

Thus, $Sigtd_I(x_n) \leq Sigtd_I(x_\ell)$, this is a contradiction.

So,

$$Sigtd_\ell(x_n) = S\delta td_1(S_G) = \sum_{i=1}^{n} SI_i$$

Also, suppose that $Sigtd_F(x_\ell) \neq S\delta td_F(S_G)$. Let x_i, $1 \geq 1 \geq n-1$ be a vertex in G such that $Sigtd_F(x_\ell) = S\delta td_F(S_G)$ and $Sigtd_F(X_n) > Sigtd_F(x_\ell)$

In addition,

$$Sigtd_F(X_i) = [\sum_{i=1}^{n} SF_{il} = Sigtd_\ell \sum_{i=l+1}^{n-1} F_{il} + SF_{nl}] + SF_\ell$$

$$\geq [\sum_{i=1}^{i-1} SF_i + (n-1)SF_i + SF_{nl}] + SF_\ell$$

$$\geq \sum_{i=1}^{n-1} SF_i + SF_i$$

$$\geq \sum_{i=1}^{n} SF_i = t\, SF_F(X_n)$$

Thus, $Sigtd_F(x_n) \leq Sigtd_F(X_\ell)$,

This is a Contradiction.

So, $Sigtd_F(X_n) = S\delta td_F(S_G) = \sum_{i=1}^{n} SF_i$ Hence the Proof.

Remark (4.1.5): -

In a complete SVNS –Graph S_G,

1. There exists at least one pair of vertices x_i and x_j such that $Sd_{T_i} = Sd_{T_j} = S\Delta_T(S_G)$, $Sd_{I_i} = Sd_{I_j} = S\delta_I(S_G)$ and $Sd_{F_i} = Sd_{I_j} = S\delta_F(S_G)$.
2. $Sigtd_T(x_i) = O_T(S_G) = S\Delta_{td_T}(S_G)$
   $Sigtd_I(x_i) = O_I(S_G) = S\delta_{td_I}(S_G)$ and
   $Sigtd_F(x_i) = O_F(S_G) = S\delta_{td_F}(S_G)$ for a vertex $x_i \in X$.
3. $\sum_{i=I}^{n} Sigtd_T(x_i) = 2 Z_T(S_G) + O_T(S_G)$,
   $\sum_{i=I}^{n} Sigtd_I(x_i) = 2 Z_I(S_G) + O_I(S_G)$ and
   $\sum_{i=I}^{n} Sigtd_F(x_i) = 2 Z_F(S_G) + O_T(S_G)$

3. CONCLUSION

In this paper, we discussed about strong degree and strong signed degree of single valued neutrosophic signed graphs. Also, we define constant SVNS-Graph, Totally constant SVNS-Graph. Strong size and strong order of SVNS-Graph is also discussed.

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